Comment on "A simple one-dimensional model of heat conduction which obeys Fourier's law"

In a recent letter Garrido et al [1] consider heat conduction in a model of hard point particles of alternating masses on a one-dimensional line. Based on a number of numerical results, the authors claim that this momentum conserving model exhibits Fourier's law. We first comment on the apparent contradiction with an earlier result of Prosen and Campbell [2] (PC). We then point out certain inconsistencies in their results and disagreements with our own results.

The authors have first measured the system size dependence of the mean current $\langle J\rangle = \langle N^{-1}\sum_l m_l u_l^3/2\rangle$ where m_l, x_l and u_l denote the mass, position, velocity of the lth particle and N is the number of particles . As they correctly point out, it is not possible to make definite conclusions from this simulation data, since the asymptotic regime may not have been attained. Next the authors compute the current-current correlation $C(t) = N\langle J(t)J(0)\rangle$ and find a decay $C(t) \sim t^{-1.3}$, which is sufficiently fast to give a finite Kubo conductivity κ. This would seem to contradict the exact result of PC on infinite thermal conductivity in momentum conserving systems. Their proof applies to this model. This apparent contradiction can be explained by the fact that, in their simulation, Garrido et al work in an ensemble with total momentum set to zero and in this case the PC proof does not predict anything. As has been pointed out by Bonetto et al [3], the correct expression for the Kubo formula requires one to use the connected part of the correlation function if one is working in the canonical ensemble [4]. Alternatively one may fix the momentum to be zero and work with the usual correlation function as Garrido et al have done. Thus the work of PC does not really prove divergence of κ.

However some aspects of the paper appear to be unsatisfactory and need some explanation. Firstly the linear temperature profiles obtained in the paper are inconsistent with the finding of finite κ. The temperature dependence of κ can be scaled out from the Kubo formula, yielding a $T^{1/2}$ dependence (given κ is finite). This follows from the fact that the correlation $C_T(t)$ at temperature T has the scaling form $C_T(t) = T^3 C_1(T^{1/2}t)$. The $T^{1/2}$ dependence of κ also follows from simple kinetic theory arguments. A temperature dependent κ at once leads to nonlinear temperature profiles. Infact in our study of the same model [5] we clearly see the expected nonlinear profiles for similar system sizes. One possible reason for the difference could be that Garrido et al use deterministic heat baths unlike the stochastic heat baths used by us. It is not clear how well such deterministic baths simulate true thermal sources. Another possibile source of error is the way Garrido et al define local temperature, namely, by measuring the mean kinetic energy and mean position of each particle. In one dimensions position fluctuations are large ($\sim \sqrt{N}$) and a more correct procedure is the one used by us: to define the local number and energy densities as $n(x,t) = \langle \sum_l \delta(x-x_l)\rangle$ and $\epsilon(x,t) = \langle \sum_l (m_l u_l^2/2)\delta(x-x_l)\rangle$ respectively, and then define the local temperature as $T(x) = 2\epsilon(x)/n(x)$.

Secondly, from our own simulations, we are unable to verify the results of [1]. The authors have computed $C(t)$ and also the onsite correlator $c(t) = \langle j_i(t)j_i(0)\rangle$. They find that for the unequal mass case, $C(t)$ and $c(t)$ have roughly the same long-time decay $\sim 1/t^{1.3}$ while for the equal mass case $c(t) \sim 1/t^3$. Our results are summarized in Fig. (1) and the important differences with [1] are:

(i) We do not find any evidence for the decay $C(t) \sim 1/t^{1.3}$. Infact the behaviour we find is consistent with the decay $J \sim 1/N^{0.83}$ found in [5]. However we do not wish to make a strong case for the number 0.83 though it is rather striking that we get the same number from two very different approaches.

(ii) The behaviour of $c(t)$ seems to be very different from that of $C(t)$ contrary to what is claimed in [1]. The authors comment that $c(t)$ has better averaging properties and, because it shows approximately the same decay, this confirms the behaviour seen for $C(t)$. But is there any reason to expect that for the unequal mass case $C(t)$ and $c(t)$ will have similar decay laws ? Infact for the equal mass case we know that $C(t)$ is a constant (since J is a constant of motion) and therefore the behaviour of $C(t)$ and $c(t)$ are drastically different.

(iii) The equal mass case is nonergodic since there are a macroscopic number of conservation laws. Thus time averages from simulations are initial-condition-dependent. We can verify this in our simulations and also find that making the masses slightly unequal restores ergodicity. Thus it is hard to understand how the decay $c(t) \sim 1/t^3$ is obtained by [1]. We note that the paper by Jepsen [6] (quoted by Garrido at al) only gives $\langle v_i(t)v_i(0)\rangle \sim 1/t^3$. It is not clear how this leads to the same prediction for the asymptotic behaviour of $c(t)$ which is more like $\langle v_i^3(t)v_i^3(0)\rangle$. Besides, the calulation of Jepsen is not done in the zero-momentum ensemble.

In our simulations time averages were performed over $10^9 - 10^{10}$ collisions. As checks on our simulations we found that $C(0)$ and $c(0)$ agree with their exactly known values and also that $C(t)$ and $c(t)$ satisfy the exact scaling relations mentioned above. In the equal mass case we got a constant $C(t)$, as expected.

In conclusion we have shown that there does not seem to be any hard evidence to prove validity of Fourier's law in this system. We think it is as hard to conclude anything (about finiteness of κ) from correlation function data as it is from the Jversus N data.

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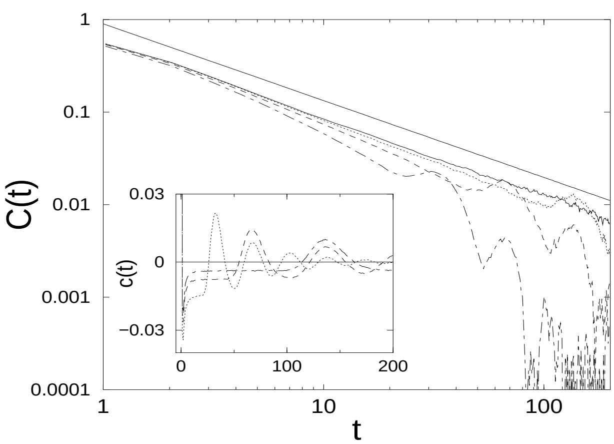

FIG. 1. Plot of $C(t)$ for system sizes $N = 100$ (dot-dashed), 200 , 400 and 800(solid) [$T = 1$, $m_1 = 1$, $m_2 = 2$]. Inset shows $c(t)$ for $N = 100$ (dots), 200 and 400 (dot-dashed) The straight line corresponds to the power-law decay $\sim 1/t^{0.83}$.